BA(P) Semester IV

Answer all questions.
Submit by 1st of April, 2017.

1) Determine the nature of the following series;

   a. $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$
   b. $\sum_{n=2}^{\infty} \frac{1}{n^2 log(n)}$
   c. $\sum\{(n^3+1)^{\frac{1}{3}} - n\}$
   d. $\frac{1^2.2^2}{1!} + \frac{2^2.3^2}{2!} + \frac{3^2.4^2}{3!} + \cdots$
   e. $\sum \frac{n!}{n^n}$
   f. $1 + \frac{2}{5}x + \frac{6}{9}x^2 + \frac{14}{17}x^3 + \cdots \ldots + \frac{2^n-2}{2^n+1}x^{n-2}$
   g. $\sum_{n=1}^{\infty} \frac{n^{n^2}}{(n+1)^{n^2}}$
   h. $\sum_{n=2}^{\infty} \frac{1}{[\log(n)]^n}$

2) Alternate Series ;
   a. Prove that every absolutely convergent series is convergent, but the converse is not true.
   b. Test the nature of the series
      i. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^p}$ $;for\ p > 0.$
      ii. $\sum_{n=2}^{\infty} (-1)^k \frac{\log k}{k^2}$
      iii. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{\sqrt{n}}$
      iv. $\sum_{k=2}^{\infty} (-1)^k \frac{1}{klog\ k}$
3) Sequence.
   a. Every convergent sequence is bounded but the converse is not true.
   b. Show that $\log_{n\to\infty} \frac{1}{n}(1 + 2^{\frac{1}{2}} + 3^{\frac{1}{3}} + \cdots . + n^{\frac{1}{n}}) = 1$
   c. Every Cauchy sequence is bounded, but the converse is not true.
   d. Prove that the sequence $\langle a_n \rangle$, where $a_n = \left(1 + \frac{1}{n}\right)^n$ is convergent and the limit lies between 2 & 3.
   e. Prove that the sequence $\langle a_n \rangle$, defined by the recursion formula $a_{n+1} = \sqrt{3a_n},\ \ a_1 = 1$ is convergent and is converges to 3.
   f. Prove that the sequence $\langle a_n \rangle$, defined by the recursion formula $a_{n+1} = \sqrt{\frac{ab^2+s_n^2}{a+1}}$ $, s_1 = a > 0, b > a$ is convergent and is converges to b.
4) Open and Closed Sets;
   a. Prove that the intersection of two nhds of a point x is also a nhd of point x.
   b. Prove that the set $s = \left\{\frac{1}{n}; n\varepsilon N\right\}$ has only one limit point namely “0”.
   c. State and prove Bolzano-Weierstrass theorem on limit.
   d. Define Limit or accumulation point of a set. Give example of each of the following and justify your answer.
      I. A set having no limit points
      II. Exactly one limit point
      III. Every point is a limit point.

e. Prove that the intersection of finite number of open set is open, but arbitrary member of open set may not be open.
f. Prove that union of finite number of closed set is closed.

5) Uniform Continuty;
   a. Every Uniform continuous function on an interval is continuous, but the converse is not true.
   b. $Prove\ that\ f(x) = x^2\ ; x\varepsilon R$ is is uniformly continuous on finite interval but not uniformally continuous on R.
   c. $show\ that\ f(x) = sinx$ is uniformly continuous on [0, infinity).
6) Riemann Integral
   a. Prove that every continuous function is integrable.
   b. If f is monotonic in [a,b], then f is integrable.
   c. Let $p = \{0,1,2,4\} be\ a\ partition\ on\ [0,4]. let\ f(x) = x^2$ then find
      I. IIpII
      II. U(p,f)
      III. L(p,f)
   d. Let $p = \left\{0, \frac{1}{3}, \frac{2}{3}, 1\right\} be\ a\ partition\ on\ [0,1]. let\ f(x) = x$ then find
      IV. IIpII
      V. U(p,f)
      VI. L(p,f)
   e. $Let\ f(x) be\ defined\ on\ [a, b] as\ follows;$
      $$f(x) = \begin{cases} 0, & x\ is\ rational \\ 1, & x\ is\ irrational \end{cases}$$
      Prove that f is not integrable.
   f. $Let\ f(x) be\ defined\ on\ [0,2] as\ follows;$
      $$f(x) = \begin{cases} x + x^2, & x\ is\ rational \\ x^2 + x^3, & x\ is\ irrational \end{cases}$$
      Prove that f is not R-integrable.
   g. If f and g are two bounded and integrable functions on [a,b] then their product fg is also bounded and integrable.
   h. If f is bounded and integrable on[a,b] the IfI is also bounded and integrable and
      $$\left|\int_a^b f(x)dx\right| \leq \int_a^b |f(x)|dx$$
   i. Let f and g are R Iintegrable over [a,b] and $f(x) \leq g(x), then\ \int_a^b f \leq \int_a^b g.$
7) Define the following.
   a. Uniform Continuty
   b. Neighbourhood of a point
   c. Open Set
   d. Closed Set
   e. R~ Integrable